Asymptotic Matching [BO] Pg 348 Sec 7.4

Intro: will be used quite often in boundary layer theory and WKB theory (ch 9 and ch 10 respectively)

Principle of asymptotic matching:

① Identify regions: the interval is broken into 2 or more overlapping subintervals.
(different behavior of x or y coupled with ε is the reason to divide our interval into different regions)

② Apply perturbation theory on each region.

③ Match those solutions in the overlapping region.

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Boundary Layer Theory [BO] Pg 431 Ch 9

Def. boundary layer: the narrow region where the solution of a differential equation changes rapidly.

Note: basically we consider perturbing parameter ε appeared as the coefficient of ODEs' higher order term.

property: as $\varepsilon \to 0$. the thickness of boundary layer should $\to 0$

Simplified situation: Now we only consider isolated (well-separated) boundary layer

Note: if that property doesn't hold, then that region is not typically boundary layer (doesn't vanish as $\varepsilon \to 0$) those solved in [BO] Ch 10, WKB

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inner region : boundary layer region.

outer region : region of slow variation of solution.

boundary layer theory : collection of perturbation methods solving differential equations whose solutions exhibit boundary layer structure.

roughly process :

① Identify boundary layers : just use ~~regular expansion~~, check: $\varepsilon = 0$ regular.
then check the solution may have crazy behavior near some points, which are the locations of boundary layers.

② for those regions far away from boundary layers are outer regions, just use regular expansion by perturbation theory.

③ for the regions near boundary layers are inner regions, these intervals are pretty small, we can take limit to simplify the problem, solve differential equation easily

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④ there will be some constants generated during the process ② and ③, so we just using our boundary conditions, as well as asymptotic matching in the overlapping regions to determine them.

⑤ the same in the process ④, asymptotic matching can form $y_{uniform} = y_{inner} + y_{outer} - y_{matching}$

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