- How to compute $y_{matching}$?

$y_{matching}$ just the double counted solution in the overlapping region. take $x=0$ boundary layer as example

compute from y_{inner} :

take $x = \delta(\varepsilon)$, rightmost of inner region. then $\varepsilon \to 0^+$

$\delta \to 0$ as $\varepsilon \to 0$ with slower rate, i.e. $\frac{\delta(\varepsilon)}{\varepsilon} \to \infty$ as $\varepsilon \to 0$

compute from y_{outer} :

again, take $x = f(\varepsilon)$, @ leftmost of outer region. then $\varepsilon \to 0$

Note : basically just let $x \to 0$ with a rate slower than $\varepsilon \to 0$

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- How to verify our final solution (uniform f) ?

  just consider variable x in inner and outer regions respectively to check whether $y_{uniform}$ is consistant with y_{inner} and y_{outer} or not.

- Case: $\varepsilon y'' + a(x)y' + b(x)y = 0 \quad y(0) = A, \; y(1) = B$.

  in conclusion, $a(x) = 0$ in $[0,1]$ is impossible.

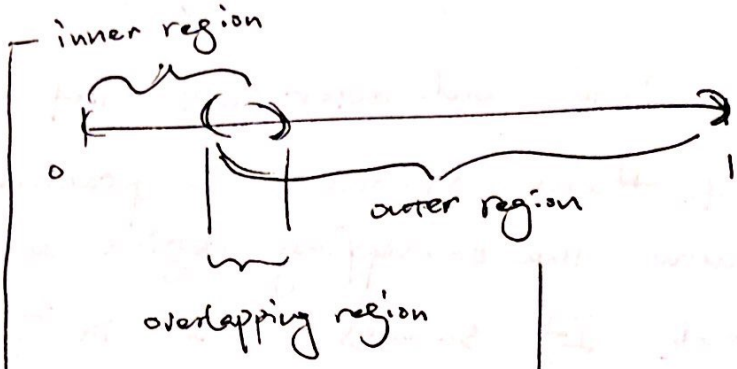
  because if so, then ~~inner~~ internal layer exist, which can be proved impossible [BO] Pg 437-438

  - $a(x) > 0$, boundary layer at $x = 0$; no boundary layer at $x = 1$
  - $a(x) < 0$, boundary layer at $x = 1$; no boundary layer at $x = 0$

  behavior of ODE determined by $a(x) > 0$ or $a(x) < 0$

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- Why We Need a New Variable ? [BO] Sec 9.2 Pg438



outer limit : $y_{out}(x) = \lim_{\varepsilon \to 0^+} y(x)$

when x outside boundary layer $\delta << x < 1$

basically just plug in $\varepsilon = 0$ in original differential equation, and just solve it.

it will have crazy behavior near boundary layer by the singularity of our given problem.

intermediate limit : $\begin{cases} x \to 0^+ & \text{slower} \\ \varepsilon \to 0^+ & \text{faster} \end{cases}$ i.e. $\bar{X} = \frac{x}{\varepsilon} \to \infty$

e.g. let $x = \varepsilon^{\frac{1}{2}} \cdot z$ with z fixed as $\varepsilon \to 0$ can get.

inner limit : $\begin{cases} x \to 0^+ & \text{the same rate as } \varepsilon \text{ or faster} \\ \varepsilon \to 0^+ & \text{not faster than } x \end{cases}$

e.g. that's why ~~for convenience~~, we use $x = \varepsilon \bar{X}$

with $\bar{X}$ fixed and finite as $\varepsilon \to 0$ can get

* inner variable $\bar{X}$ and intermediate variable z are better describing the function behavior in their own regions. that's why we need to define new variables to replace x.

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- How to check the existence of overlapping region ?

  formally, expanding inner and outer into infinite series, and check them. However, in practice, we often just assume the overlapping region exist, then do the process. If success, then it just how as we described.

- other need to consider: even we achieve ∞-order precision, our approximated solution may still not the exact solution. There can be an exponentially small error of order $e^{-\frac{1}{\varepsilon}}$.

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